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Assignment 7: MTH 213, Fall 2017

Ayman Badawi

QUESTION 1. Let $a, b, c, d \in R$, where $a < b$ and $c < d$. Prove that $|[a, b]| = |[c, d]|$

Hint construct a bijective function f from $(-\infty, 0]$ onto $(a, b]$, for example let $f(x) = (b - a)e^x + a$. Construct another bijective function L from $(-\infty, 0]$ onto $(c, d]$. What is L? Convince yourself that f, L are indeed bijective functions (draw them !) now it is clear using some facts (may be some how you can add the missing a and the missing c

QUESTION 2. Let $A = \{x, 6, 9, y, 2\}$. Define "=" on $P(A)$: whenever $a, b \in P(A)$, then $a = b$ iff $|a| = |b|$.
Show that "=" is an equivalence relation and find all equivalence classes.

QUESTION 3. Define "=" on Z: whenever $a, b \in Z$, then $a = b$ iff $a \mid b$ (i.e., a is a factor of b). Show that "=" is not an equivalence relation

QUESTION 4. Let $A = \{2, 3, 4, 8, 9, 15, 17, 22\}$, $B = \{0, 1, 2\}$. Define "=" on A: whenever $a, b \in A$, then $a = b$ iff $|a - b| \in B$. Is "=" an equivalence relation. If yes, explain, then convince me and find all equivalence classes.

QUESTION 5. Define "=" on Q : whenever $a, b \in Q$, then $a = b$ iff $a - b \in Z$. Convince me that "=" is an equivalence relation and describe all equivalence classes (note that $a = b$ iff $a = b + x$ for some $x \in Z$)

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

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Q1. $A = \{x, 6, 9, y, 2\}$

$a, b \in P(A)$, $a = b$ iff $|a| = |b|$
show "$=$" is an eq rel. & find all eq. classes

$P(A) = \{\{x\}, \{6\}, \{9\}, \{y\}, \{2\}, \{x, 6\}, \ldots$
$\ldots, \{x, 6, 9\}, \ldots, \{x, 6, 9, y\}, \ldots$
$\ldots, \{x, 6, 9, y, 2\}, \{\emptyset\}\}$.

① symmetric: $\forall\ a \in P(A)$

$|a| = |a|$ thus $a = a$

② Ref Assume $a = b$ for some $a, b \in P(A)$. Show $b = a$
since $a = b$ we have $|a| = |b|$

thus $|b| = |a|$ $\quad |\{6\}| = |\{x\}|$

Hence $b = a$.

③ Transistive. Assume $a = b$ & $b = c$ for some $a, b, c \in P(A)$
Show $a = c$

we know $|a| = |b|$
we know $|b| = |c|$

Hence $|a| = |c|$

Eq. classes.

$[\{x\}] = \{\{x\}, \{6\}, \{9\}, \{y\}, \{2\}\}$

$[\{\emptyset\}] = \{\{\emptyset\}\}$

$[\{x,6\}] = \{\{x,6\}, \{x,9\},$

$\{x,y\}, \{x,2\}, \{6,9\},$

$\{6,2\}, \{6,y\}, \{9,y\}$

$\{2,9\}, \{2,y\}\}$

$[\{x,6,9\}] = \{\{x,6,9\}, \{x,6,2\}$

$\ldots\ldots\}$

$\vdots$

Q4 $A = \{2,3,4,8,9,15,17,20\}$
$B = \{0,1,2\}$

Def "=" on A
$a, b \in A$, $a = b$ iff $|a-b| \in B$
Is "=" an eq rel.?

① Symmetric: $\forall\ a \in A$

$|a-a| \in B$ thus $a = a$

② Ref. Assume $a = b$ for some $a, b \in A$ show $b = a$

Since $a = b$, $|a-b| \in B$
thus $|b-a| \in B$

Hence $b = a$

③ Trans. Assume $a = b$ & $b = c$ for some $a, b, c \in A$
Show $a = c$
take $a = 2$, $b = 3$, $c = 4$
$|a-b| + |b-c| = |a-c|$
$= |2-3| + |3-4| = |2-4|$

$2 = 2$ ✓

eq. classes:

$[2] = \{2, 3, 4\}$

~~same~~

~~[3]~~

~~same~~

~~[4]~~

$[8] = \{8, 9\}$

$[15] = \{15, 17\}$

$[22] = \{22\}$

<u>QS</u> Def "=" $\mathbb{Q}$

$a, b \in \mathbb{Q}$, $a = b$ iff $a - b \in \mathbb{Z}$

($a = b$) iff $a = b + x$ for some $x \in \mathbb{Z}$

① symm. : $\forall\, a \in \mathbb{Q}$

$a - a \in \mathbb{Z}$ thus $a = a$

② Ref. Assume $a = b$ for some $a, b \in \mathbb{Q}$, show $b = a$

$a = b$, $a - b \in \mathbb{Z}$

thus $b - a \in \mathbb{Z}$

Hence $b = a$

③ Trans.

we know $a - b \in \mathbb{Z}$

" " $b - c \in \mathbb{Z}$

$(a - b) + (b - c) \in \mathbb{Z}$

$a - c \in \mathbb{Z}$

$a = c$

eq. classes

$\overline{0} = \mathbb{Z} = \{\ldots, -3, -2, -1, 0,$
$1, 2, 3, \ldots\}$

$\overline{\frac{1}{2}} = \frac{1}{2} + \mathbb{Z} \; \{\ldots, -2.5, -1.5,$
$-0.5, 0.5, 1.5 \ldots\}$

$\overline{\frac{1}{3}} = \frac{1}{3} + \mathbb{Z}$

$\overline{\frac{2}{3}} = \frac{2}{3} + \mathbb{Z}$

$\overline{\frac{1}{4}} = \frac{1}{4} + \mathbb{Z}$

$\overline{\frac{3}{4}} = \frac{3}{4} + \mathbb{Z}$

$\vdots$

In general, let $n \in \mathbb{N}^*$
$n \geq 5$ (because we did until 4 previously)

$\frac{a}{n} + \mathbb{Z}$, $\gcd(a, n) = 1$
and $1 \leq a < n$

$\Rightarrow$ if $a = n$, then you go back to the first class.

Q3. Def. "$=$" on $\mathbb{Z}$

$a, b \in \mathbb{Z}$, $a = b$ iff $a \mid b$

Show that "$=$" is not an eq rel

symmetric: $\forall \; a \in \mathbb{Z}$

$a \mid a$ thus $a = a$

reflexive: assume $a = b$ for some $a, b \in \mathbb{Z}$, show $b = a$

Since $a = b$ we have $a \mid b$ but is $b \mid a$?

Hence $b \neq a$

$\therefore$ "$=$" is not an equivalence relation.

let $a = 3$, $b = 6$

since $3 \mid 6$, $a = b$

but $6 \nmid 3$, $\therefore$ $b \neq a$.